Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a})$ $\qquad$ $\int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da}$ $\qquad$ $\int (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$

Cartesian Coordinates: $\quad \overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z} \qquad d\tau = dx\, dy\, dz$

$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$

Spherical Coordinates: $\quad x = r\, sin\theta\, cos\phi,\ y = r\, sin\theta\, sin\phi,\ z = r\, cos\theta$

$\overrightarrow{dl} = dr\hat{r} + r\, d\theta\hat{\theta} + r\, sin\theta\, d\phi\hat{\phi} \qquad d\tau = r^2 sin\theta\, dr\, d\theta\, d\phi$

$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\, sin\theta}\frac{\partial (sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\, sin\theta}\frac{\partial A_\phi}{\partial \phi}$

$\nabla \times \vec{A} = \frac{1}{r\, sin\theta}\left(\frac{\partial (sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r\, A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\phi}$

Cylindrical Coordinates: $\quad x = s\, cos\phi,\ y = s\, sin\phi,\ z = z$

$\overrightarrow{dl} = ds\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z} \qquad d\tau = s\, ds\, d\phi\, dz$

$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial (s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

$\nabla \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$

Dirac Delta Function: $\int \delta^3(\vec{r} - \vec{a})\, d\tau = 1$ if $\vec{a}$ contained in volume, $\delta^3(\overrightarrow{\Delta r}) = \frac{1}{4\pi}\nabla \cdot \left(\frac{\widehat{\Delta r}}{\Delta r^2}\right)$

Irrotational Functions: $\quad \nabla \times \vec{A} = 0 \qquad \vec{A} = \nabla f \qquad \oint \vec{A} \cdot \overrightarrow{dl} = 0$

Solenoidal Functions: $\quad \nabla \cdot \vec{F} = 0 \qquad \vec{F} = \nabla \times \vec{A} \qquad \oint \vec{F} \cdot \overrightarrow{da} = 0$

Electric Field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\, d\tau'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\, da'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\, dl'$

Gauss's Law: $\oint \vec{E}\cdot\overrightarrow{da} = Q_{enc}/\varepsilon_0$, $\nabla\cdot\vec{E} = \rho/\varepsilon_0$

Electric Potential: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r\prime})}{\Delta r} d\tau'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r\prime})}{\Delta r} da'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r\prime})}{\Delta r} dl'$
$\vec{E} = -\nabla V$, $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot\overrightarrow{dl}$, $\nabla^2 V = -\rho/\varepsilon_0$

Boundary Conditions: $\Delta\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, $\Delta V = 0$

Work and Energy: $W = Q\Delta V$, $W = \frac{\varepsilon_0}{2}\int E^2 d\tau$